0166

Total No. of Questions - **30**
Total No. of Printed Pages - 3

Regd. No.

Part - III

MATHEMATICS, Paper - IA

(English Version)

MODEL QUESTION PAPER (FOR IPE 2020-21 ONLY)

***Time :* 3 Hours** **_Max. Marks :_ 75**

Note: This question paper consists of three section A, B and C.

Section - A

Very short answer type questions.

(i) Answer all questions.

(ii) Each question carries 2 marks. **10×2=20**

1. If $A=\left\{0,\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2}\right\}$ and F : A→B is a surjection defined by $f(x) = \cos x$, then find B.

2. Find the domain of the real valued function $f(x)=\frac{1}{\log(2-x)}$.

3. If $A=\begin{bmatrix}2 & 3 & -1\\ 7 & 8 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 1\\ 2 & -4 & -1\end{bmatrix}$ then find A+B.

4. If $A=\begin{bmatrix}i & 0\\ 0 & i\end{bmatrix}$, find A^2.

5. if $\bar{a}=2\bar{i}+5\bar{j}+\bar{k}$ and $\bar{b}=4\bar{i}+m\bar{j}+n\bar{k}$ are collinear, then find m and n.

6. Find the vector equation of the line passing through the point $2\bar{i}+3\bar{j}+\bar{k}$ and parallel to the vector $4\bar{i}-2\bar{j}+3\bar{k}$.

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7. If $\bar{a} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{b} = 3\bar{i} - \bar{j} + 2\bar{k}$ then show that $\bar{a} + \bar{b}$ and $\bar{a} - \bar{b}$ are perpendicular to each other.

8. Prove that $\dfrac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$.

9. Find the period of the function defined by $f(x) = \tan\left(x + 4x + 9x + ... + n^2 x\right)$.

10. If sinh$x$ = 3, then show that $x = \log_e\left(3 + \sqrt{10}\right)$.

Section - B

Short answer type questions. **5×4=20**

(i) Answer any FIVE questions.

(ii) Each question carries four marks.

11. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then show that $(aI + bE)^3 = a^3I + 3a^2bE$ where 'I' is unit matrix of order 2.

12. Show that $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ is non-singular and find A^{-1}.

13. Let ABCDEF be regular hexagone with centre O, show that $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\overline{AD} = 6\overline{AO}$.

14. Find the equation of the plane passing through the point $\bar{a} = 2\bar{i} + 3\bar{j} - \bar{k}$ and perpendicular to the vector $3\bar{i} - 2\bar{j} - 2\bar{k}$ and the distance of this plane from the origin.

15. If the vectors $\bar{a} = 2\bar{i} - \bar{j} + \bar{k}$, $\bar{b} = \bar{i} + 2\bar{j} - 3\bar{k}$ and $\bar{c} = 3\bar{i} + p\bar{j} + 5\bar{k}$ are coplanar, then find 'P'.

16. If A is not an integral multiple of $\frac{\pi}{2}$, then prove that

    (i) tanA + cotA = 2 cosec2A

    (ii) cotA – tanA = 2 cot2A

17. Find the range of $7\cos x - 24\sin x + 5$.

18. Prove that $\dfrac{\cosh x}{1 - \tanh x} + \dfrac{\sinh x}{1 - \coth x} = \sinh x + \coth x$ for $x \neq 0$.

19. Prove that $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\frac{s^2}{\Delta}$.

20. If $\sin\theta=\frac{a}{b+c}$ then show that $\cos\theta=\frac{2\sqrt{bc}}{b+c}\cos\frac{A}{2}$.

Section - C

Long Answer type questions. **5×7=35**

(i) Answer any FIVE questions.

(ii) Each question carries seven marks.

21. If $f=\{(1, 2), (2, -3), (3, -1)\}$ then find (i) 2f (ii) 2+f (iii) f^2 (iv) $\sqrt{f}$

22. If $A=\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$, then find A^3-3A^2-A-3I, where I is unit matrix of order 3.

23. Solve the following system of equations by Cramer's rule

$x+y+z=1,\ \ 2x+2y+3z=6,\ x+4y+9z=3.$

24. Solve the following system of equations by Matrix Inversion method

$2x-y+3z=9,\ \ x+y+z=6,\ x-y+z=2.$

25. Find the vector equation of the plane passing through points $4\bar{i}-3\bar{j}-\bar{k}$, $3\bar{i}+7\bar{j}-10\bar{k}$ and $2\bar{i}+5\bar{j}-7\bar{k}$ and show that the point $\bar{i}+2\bar{j}-3\bar{k}$ lies in the plane.

26. If $\bar{a}=7\bar{i}-2\bar{j}+3\bar{k}$, $\bar{b}=2\bar{i}+8\bar{k}$ and $\bar{c}=\bar{i}+\bar{j}+\bar{k}$, then compute $\bar{a}\times\bar{b}$, $\bar{a}\times\bar{c}$ and $\bar{a}\times(\bar{b}+\bar{c})$. Verify whether the cross product is distributive over vector addition.

27. If [b c d] + [c a d] + [a b d] = [a b c]. Then show that the points with position vectors a, b, c and d are coplanar.

28. If A, B, C are angles in a triangle, then prove that

sinA + sinB – sinC = $4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$.

29. If $\cot\frac{A}{2}:\cot\frac{B}{2}:\cot\frac{C}{2}=3:5:7$, then show that a:b:c = 6:5:4.

30. If a = 13, b = 14, c = 15, show that $R=\frac{65}{8}$, $r=4$, $r_1=\frac{21}{2}$, $r_2=12$ and r_3 = 14.

Turn Over